How to improve predictions of future WAIS behavior

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We thank the NSF, which has supported the development of this model over many years through several different grants.
Modeling ice sheets has proceeded from:

- The very simplest parabolic profiles of the perfectly plastic approximation, a 1-D, steady-state solution where the driving stress is exactly balanced by a uniform basal yield stress;

\[ h(x) = \sqrt{\frac{2\tau x}{\rho g}} \]
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- On to the elliptical profiles, where a uniform accumulation rate is assumed, again a 1-D steady-state solution, but now the basal stress varies along the flowline;

\[
h(x) = 2^{\frac{3}{8}} \left( \frac{5 \dot{a}}{2} \right)^{\frac{1}{8}} \left( \frac{A}{\rho g} \right)^{\frac{3}{8}} \left[ L^{\frac{4}{3}} - (L - x)^{\frac{4}{3}} \right]^{\frac{3}{8}}
\]

Elliptical: SW Greenland

[Graph showing data for SW Greenland with A=3, \( \dot{a}=1.3 \)]
Modeling ice sheets has proceeded from:

- To the shallow-ice approximation, a 1-D flowline or 2-D map-plane, time-dependent model, where only the basal stress is included and is assumed to equal the driving stress;

\[ \nabla \cdot (-k(x, y) \nabla h) = \dot{a}(x, y) - \frac{\partial h}{\partial t} \]

\[ k(x, y) = -\left( \left[ \frac{\rho g}{B} \right]^m H^{m+1} |\nabla h|^{m-1} w^q + \left[ \frac{2}{n+2} \right] \left[ \frac{\rho g}{A} \right]^n H^{n+2} |\nabla h|^{n-1} \right) \]
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Modeling ice sheets has proceeded from:

- To the "shelfy-flow" models which are ad hoc adaptations of Morland's ice shelf equations, a 1-D or 2-D time-dependent solution with only longitudinal stresses and an added basal resistance term;

\[
\frac{\partial}{\partial x} \left( 2 \bar{\nu} h \left( 2 \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right) + \frac{\partial}{\partial y} \left( \bar{\nu} h \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right) = \rho g h \frac{\partial z_s}{\partial x} - \tau_x
\]

(3.63)

\[
\frac{\partial}{\partial y} \left( 2 \bar{\nu} h \left( 2 \frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} \right) \right) + \frac{\partial}{\partial x} \left( \bar{\nu} h \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right) = \rho g h \frac{\partial z_s}{\partial y} - \tau_y
\]

(3.64)

\[
\bar{\nu} = \frac{1}{h} \int_{z_b}^{z_s} \frac{B(T(z)) \, dz}{2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \frac{1}{4} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \frac{\partial u \, \partial v}{\partial x \, \partial y} \right]^{\frac{n-1}{2n}}}
\]

(3.7)
Modeling ice sheets has proceeded from:

- To the current full-momentum solvers, true 3-D, time-dependent solutions where all stresses are presumably accounted for.
The Full Momentum Equation

- Conservation of Momentum: Balance of Forces
- Flow Law, relating stress and strain rates.
- Effective viscosity, a function of the strain invariant.

\[
\sigma_{ij,j} + \rho a_i = 0 \quad (1)
\]

\[
\sigma_{ij} = \delta_{ij}P + 2\mu \dot{\varepsilon}_{ij} \quad (2)
\]

\[
2\mu = B \dot{\varepsilon} \frac{1-n}{n} \quad (3)
\]
The Full Momentum Equation

- The strain invariant.
- Strain rates and velocity gradients.
- The differential equation from combining the conservation law and the flow law.

\[ \dot{\epsilon}^2 = \frac{1}{2} \dot{\epsilon}_{ij} \dot{\epsilon}_{ij} \]  
(4)

\[ \dot{\epsilon}_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \]  
(5)

\[ (\delta_{ij} P + 2\mu \frac{1}{2} (u_{i,j} + u_{j,i}))_{,j} + \rho a_i = 0 \]  
(6)
What do these all have in common?

- FUDGE FACTORS !!!
What do these all have in common?

• err....

• PARAMETERS . . .
Modeling ice sheets has proceeded from:

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Modeling ice sheets has proceeded from:

- On to the elliptical profiles, where a uniform accumulation rate is assumed, again a 1-D steady-state solution, but now the basal stress varies along the flowline;

\[ h(x) = 2^{3/8} \left( \frac{5a}{2} \right)^{1/8} \left( \frac{A}{\rho g} \right)^{3/8} \left[ L^{4/3} - (L - x)^{4/3} \right]^{3/8} \]
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\frac{\partial}{\partial x} \left( 2\bar{\nu}h \left( 2\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right) + \frac{\partial}{\partial y} \left( \bar{\nu}h \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right) = \rho gh \frac{\partial z_s}{\partial x} - \tau_x
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(3.63)

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(3.64)

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\[ 2\mu = B\dot{\varepsilon}^{\frac{1-n}{n}} \quad (3) \]
What do these all have in common?

- PARAMETERS that quantify
  - THAT WHICH WE DON’T KNOW
  - ABOUT THE PHYSICS . . .
What do these all have in common?

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\frac{\partial}{\partial y} \left( 2\bar{v} h \left( 2\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} \right) \right) + \frac{\partial}{\partial x} \left( \bar{v} h \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right) = \rho gh \frac{\partial z_s}{\partial y} - \tau_y \tag{3.64}
\]

\[
\bar{v} = \frac{1}{h} \int_{z_b}^{z_g} \frac{B(Y(z)) \, dz}{2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \frac{1}{4} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} \right]^{\frac{n-1}{2n}}} \tag{3.7}
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\[
2\mu = B \varepsilon \frac{1-n}{n}
\]  \hspace{1cm} (3)
What do these all have in common?

- **PARAMETERS** obtained by
  - TUNING the model to DATA.

Parabolic: SW. Greenland

data parabolic $\tau = 1.6$
What do these all have in common?

- PARAMETERS obtained by TUNING the model to DATA.

Elliptical: SW Greenland

\[ A = 3, \dot{a} = 1.3 \]
What do these all have in common?

- PARAMETERS obtained by tuning the model to DATA.
Improvements in the models

- Thermo-mechanical are “better”
  - Because we no longer need to “specify” the ice hardness.

\[ B(T) = B_0 \exp\left(-\frac{Q}{RT}\right) \]

- BUT we now include a

- “flow enhancement factor” to allow us to still “tune” the model.
- A parameter that accounts for that which we do not know (fabric history? impurity content?)
Improvements in the models

- Explicit inclusion of SLIDING

\[ k(x, y) = - \left( \left[ \frac{\rho g}{B} \right]^m H^{m+1} |\nabla h|^{m-1} w^q + \frac{2}{n + 2} \left[ \frac{\rho g}{A} \right]^n H^{n+2} |\nabla h|^{n-1} \right) \]
Improvements in the models

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- with a “lubricating factor” to turn on and off the fast flow mechanism.
- We expect it depends on the presence of water, but is it different?
  - for hard-rock vs deformable bed sliding? conduit vs film? lakes vs wet bed?
Other models that are “tuned”

- NCEP/NCAR Re-analysis Project
  - 1948 to present gridded dataset
  - GCM constrained by observations: Land stations
Other models that are “tuned”

- NCEP/NCAR Re-analysis Project
  - 1948 to present gridded dataset
  - GCM constrained by observations:

Aircraft
Other models that are “tuned”

- NCEP/NCAR Re-analysis Project
  - 1948 to present gridded dataset
  - GCM constrained by observations: Ships
Other models that are “tuned”

- NCEP/NCAR Re-analysis Project
  - 1948 to present gridded dataset
  - GCM constrained by observations: Sondes
Other models that are “tuned”

- NCEP/NCAR Re-analysis Project
  - 1948 to present gridded dataset
  - GCM constrained by observations: Satellite temps
Other models that are “tuned”

- NCEP/NCAR Re-analysis Project
  - 1948 to present gridded dataset
  - GCM constrained by observations:
    - Satellite winds

Satellite winds
Other models that are “tuned”

- The kinds of questions they are asking:

3.2 Diagnosing the differences
3.2.1 Are wrong type of clouds being formed?

Fig. 4. Scatter plot of solar cloud forcing (x-axis) vs. longwave cloud forcing (y-axis) for each latitudinal band, using monthly averaged buoy data (black), ECMWF data (blue), NCEP2 values (magenta), and ISCCP (red circles).

For Full Time Period

For Precipitating Clouds

For Non-Precipitating Clouds

Results:
- NCEP2 does not capture cold tongue near 1-1 relation. Instead, NCEP2 tends to produce ITCZ type clouds in cold tongue region.
- There was no significant rainy months in the stratus deck region, and no significant dry months in the frontal region.
- Slope in southern region during Rainy months is similar to slope in ITCZ region -- ITCZ radiative properties are similar, whether in NH or SH. (although range of cloud forcing is less in sh ITCZ than in nh ITCZ).
- In the southern region, the slope ICFRL/CFRSL increased during dry periods. This change in radiative properties was not captured by NCEP2.
The Time-Dependent Problem

- The “spin-up” problem...
- We need a good starting point from which to project into the future.
- We are not starting from a simple steady-state ice sheet,
  - but instead from one that has undergone significant changes in the not too distant past.
Most geological reconstructions of the ice sheet have the Ross Sea beneath the current ice shelf fully grounded, possibly out to the continental shelf, with the major retreat occurring relatively late.
The Time-Dependent Problem

• With such a recent major change in the ice sheet configuration, major features such as the internal temperature field and the distribution of water at the bed will have preserved in them transient features reflective of the retreat history.

• Both internal temperature and basal water have a strong impact on the ice sheet's dynamic behavior, and hence must be well characterized in order to have a good starting point for a predictive model.
The Time-Dependent Problem

- The easiest way to accommodate this is to run the model for a glacial cycle capturing the endpoint as initial conditions for the predictive run.
- One problem with this approach is that the known history of the ice sheet is relatively short, and not unambiguously understood.
The Time-Dependent Problem

- The expanded extent of the ice sheet,
- When it stood at that larger configuration,
- The increase in volume,
- The timing of the recent collapse
  - are all controversial questions.
- Another problem of course involves which climate "proxy" to use to drive the ice sheet through its cycle, and how to couple that proxy to the controlling mechanisms.
Conclusions

• Expecting higher-order models to solve all our problems is naive.
  
  – Higher-order models contain constants and parameters which are not well known too.

• We need a clearer understanding of the physical processes involved,

• or at least an adequate and versatile parameterization of said process that can be tuned to match the current configuration.

• Included in this of course is the recognition that we are dealing with a time-dependent creature,
  
  – one whose current configuration (the starting point for our predictions) contains transients reflecting recent past behavior, which must be well characterized
  
  – We need unambiguous DATA.
THICKNESS [7041]
TIME = 205
THICKNESS [7061]
TIME = 305

THICKNESS [1561]
TIME = 305
THICKNESS [7161]
TIME= 805

THICKNESS [1661]
TIME= 805
THICKNESS [7221]
TIME= 1105
THICKNESS [7261]
TIME = 1305
THANK YOU
THANK YOU
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