Grounding line migration in a two-dimensional marine ice stream model

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Motivation

- Grounding line position and stress transmission across grounding line have been shown to be important factors in the dynamics of marine ice streams (*Schmeltz et al 2002, Payne et al 2004*).
- Numerical simulation of grounding line migration has proven difficult (*Vieli & Payne, 2005*); high resolution may be necessary to adequately resolve stress in the transition zone (*Schoof, 2007*).

Goals

- Our goal is to develop a numerical model of an ice stream-ice shelf system that produces **robust** solutions and allows us to examine the role of ice shelf buttressing and ice shelf geometry in marine ice stream dynamics.
- By **robust** we mean with respect to grid resolution, details of discretization, and initial conditions.

Shelfy-Stream Model (MacAyeal, 1989)

$$\nabla \cdot (h\nu\vec{D}) + \vec{\tau_b} = \rho g h \nabla s, \quad D_{ij} = 2\dot{\epsilon}_{ij} + 2(\dot{\epsilon}_{xx} + \dot{\epsilon}_{yy})\delta_{ij},$$

$$\nu = \frac{B}{2}\left(u_x^2 + v_y^2 + u_x v_y + \frac{1}{2}(u_y + v_x)^2\right)^{-1/3},$$

$$h_t + \nabla \cdot (\vec{u}h) = a,$$

where $\vec{u} \equiv (u, v)$ is horizontal velocity, h is thickness, s is surface elev, a is mass balance, B is a constant, and $i, j \in (1, 2)$.

Basal stress parameterization:

$$\vec{\tau}_b = -C|u|^{m-1}\vec{u}, \quad m = \frac{1}{3}, \; C = \text{constant}$$

where ice is **grounded**, i.e. where $h > \frac{\rho_w}{\rho} z_{bedrock}$ (the *flotation condition*).

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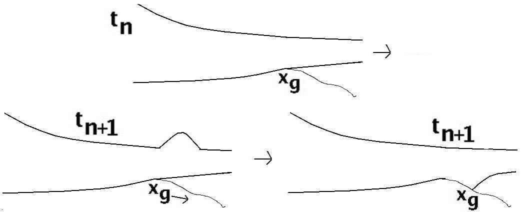
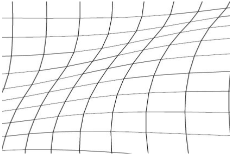
Numerics

- Diagnostic equation for u solved by a finite element method with continuous, piecewise bilinear nodal basis functions on quadrilateral cells
- Prognostic equation for h solved by finite volume method, treating h as piecewise constant
- Grounding line movement - diagnosed within eulerian framework ("fixed grid" of *VP05*) rather than ALE method ("moving grid" of *VP05*)

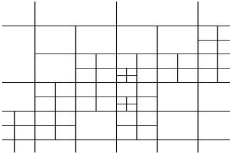


Mesh Adaptation (2 Methods)

- *r*-**refinement** (moving mesh) - gridpoints moved, connectivity and # of cells remain constant



- *h*-**refinement** - dividing and merging of cells - "hanging node" issues

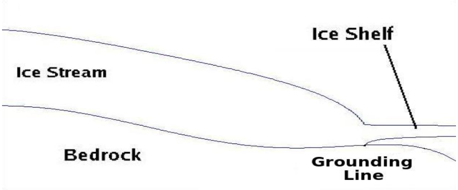


Model, cont'd

Model timestep algorithm:

1. Evolve the thickness (h) from time t_n to t_{n+1} with the velocities calculated at t_n
2. Move (or refine and coarsen) the mesh
3. Perform high-order conservative interpolation of h on to new mesh
4. Diagnose position of grounding line from $h(t_{n+1})$
5. Solve diagnostic equations for velocity
6. repeat..

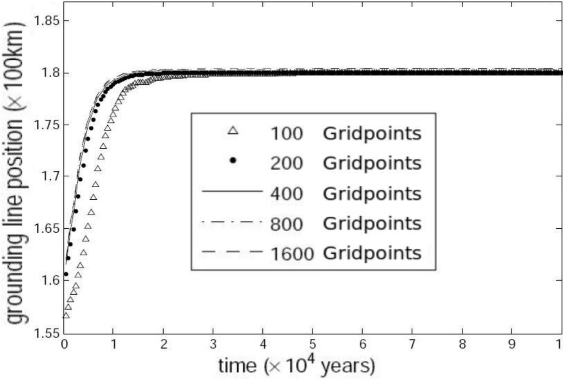
1D Excursion



The idea was to first investigate a one-dimensional (y-independent) version of the Shelfy-Stream model before taking on the 2D world. A 1D model has some advantages:

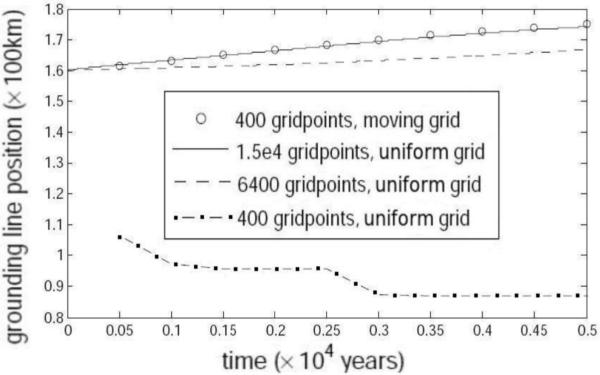
- Much easier to check convergence, robustness w.r.t. discretization
- Results can be checked against other studies (e.g. *Schoof, 2007*; Eulerian vs ALE)

1D Convergence



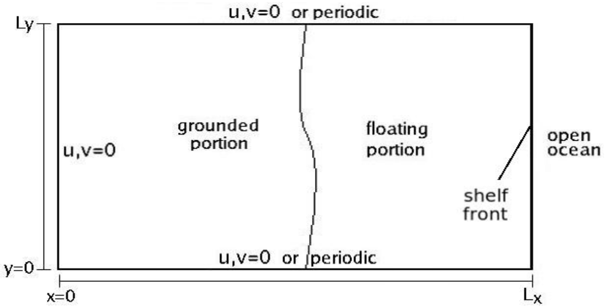
Grounding line position (x_g) versus time for a moving mesh simulation, convergence study (no *h*-refined results).

1D results - Need for adaptive mesh



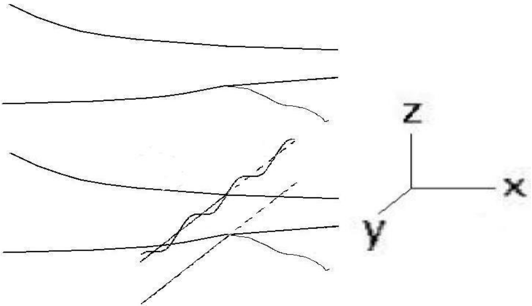
Agreement with ultra-high resolution uniform mesh; low-resolution uniform mesh is qualitatively different

2D Domain



2D model domain: either no-slip or periodic side boundaries. The entire domain is ice-covered and bedrock is y-independent.

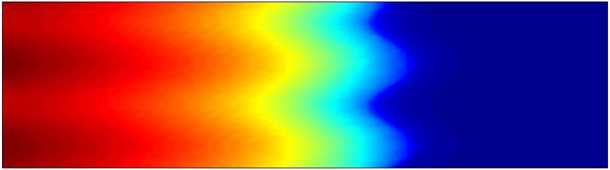
Stability of grounding line?



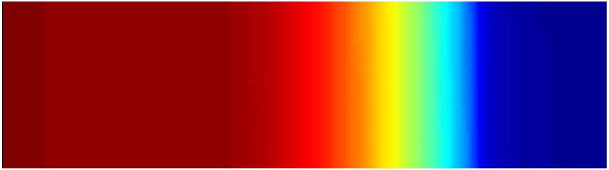
Can we expect to recover “1D” solutions with a 2D model?

Stability of grounding line?

Initial condition:



Steady State:



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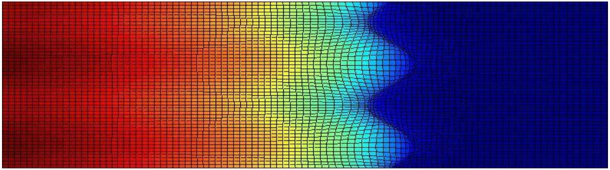

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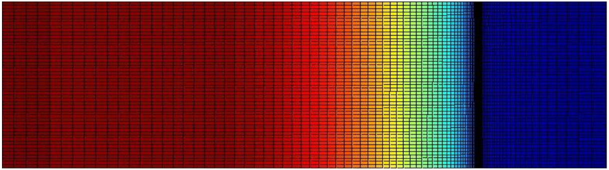
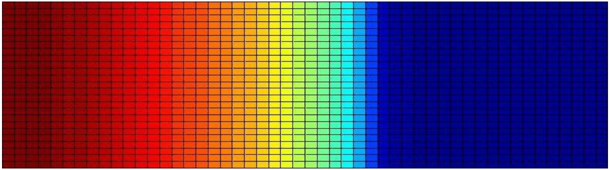


Need for adaptive mesh - 2D

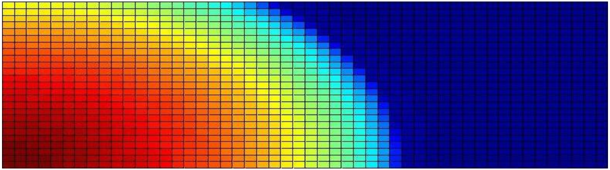
Can an adaptive mesh model yield robust results? Is adaptive refinement necessary?

Experiments, Uniform Mesh

Initial Condition 1:



Initial Condition 2:



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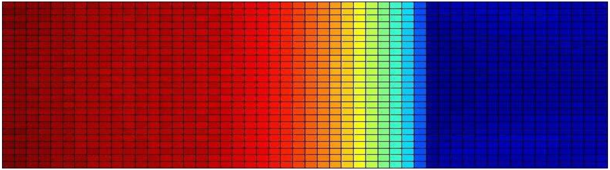

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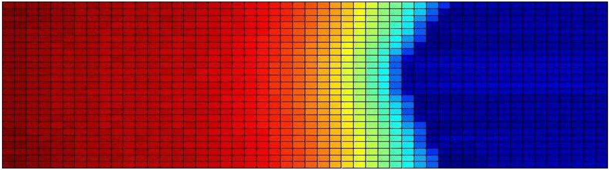

Experiments, Moving Mesh

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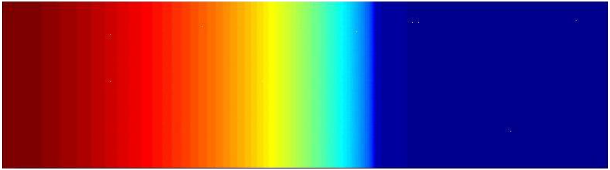

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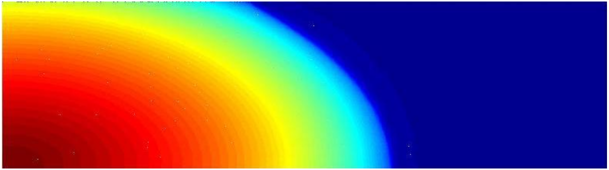

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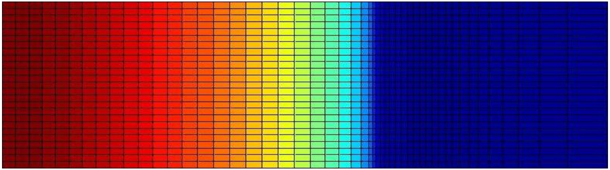

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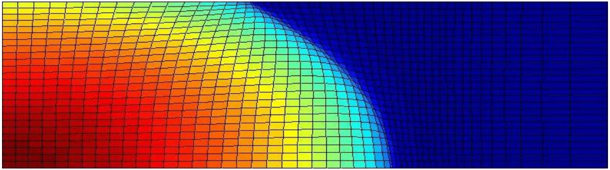

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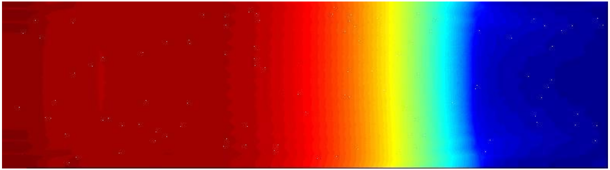

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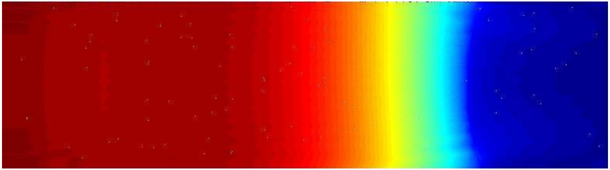

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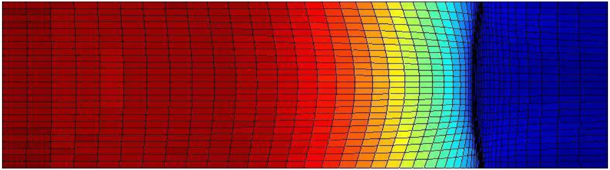

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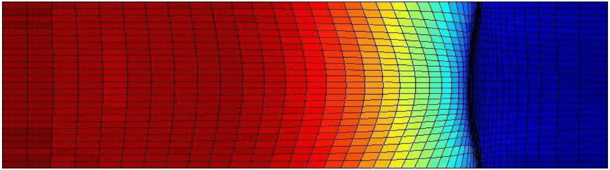
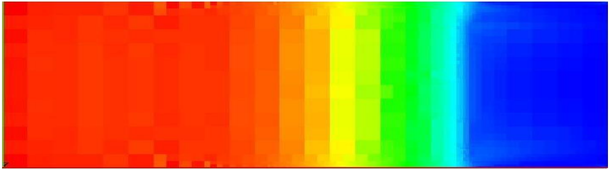
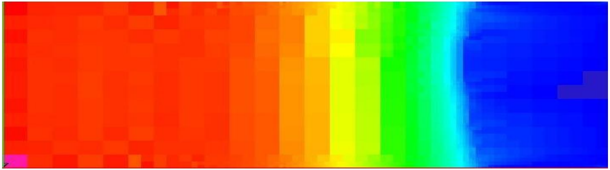


Experiments, Refined Mesh

Steady State 1 (Initial Condition same as previous):



Steady State 2 (Initial Condition same as previous):



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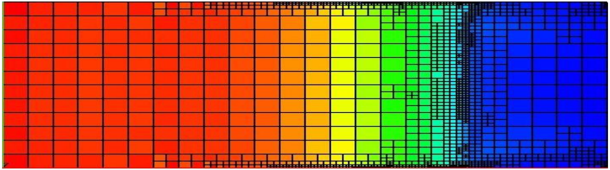

Steady State 2 (Initial Condition same as previous):