Control of the width of West Antarctica ice streams by internal melting in ice sheet at the margins

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Introduction

West Antarctica Ice streams (WAIS) characteristics:
- Some tens of km wide (40 to 80 km)
- Some hundreds of km long
- $V$ from 100 to 800 m.yr$^{-1}$

Fig.: Ross Sea ice streams. Interferometric synthetic aperture radar (InSAR) velocity (from Joughin et al., 2002) overlaid on a hillshade of the surface digital elevation model (Bamber et al., 2009). Velocity contours shown are 25 m/a (thin line) and 250 m/a (thick line).

Modified from [Le Brocq, Payne, Siegert & Alley, J. Glac., 2009]
Field evidence of channel at margins

Fig.: Clarke et al. [2000] have used high-power radar system at now abandoned shear margin and have found bottom diffractors.
They argue that at least some of the diffractions originate from entrained *morainal debris* at the base of recently abandoned shear margin of ice stream B.

→ One of the possible mechanisms is meltwater processes depositing sediment in a subglacial stream.
Field evidence of channel at margins

Fig.: Borehole observation at the shear margin of ice stream C shows a 1.6 m tall water-filled cavity. Video of the borehole shows horizontal acceleration of particles sinking into the cavity, indicating flow of water within the cavity -- part of channel?
Modified from [Vogel PhD Thesis, 2004] and [Vogel et al., GRL, 2005]
Field evidence of internal melting at margins

Clarke et al. [2000], in order to explain the bottom diffractors, have also invoked partial melting in temperate ice to a height of \( \approx 230 \) m that developed due to strain heating.
Open question

(Oz) Ice flow direction

(not to scale; b ~ 40-80 H)

Fig.: Cross section of a West Antarctica ice stream.

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Open question

Could the 40 to 80 km widths of West Antarctic ice streams be controlled by onset of melting within the ice sheet at the stream margins?
Outline

• Evidence of internal melting of the ice sheet at stream margins in West Antarctica

• How meltwater rains downward? What water fluxes are involved? What are the possible consequences of this drainage for fast-flowing ice streams?
1/ Evidence of internal melting

\[ \tau_{\text{grav}} = \rho g HS \]
\[ = \text{downslope gravity force per unit base area} \]
\[ \tau_{\text{lat}} H = (\tau_{\text{grav}} - \tau_{\text{base}})_{\text{avg}} x \]

(neglecting any variation in net axial force in sheet, as justified by Whillans and van der Veen [J. Glac. 1993])
Heating source at WAIS margins

\[ \tau_{lat} H = (\tau_{grav} - \tau_{base})_{avg} x \]

- \( \tau_{lat} \) develops and increases \( \sim \) linearly with distance \( x \).
- Rheological creep law for ice, Glen’s law: \( \dot{\gamma}_{lat} = 2B^{-3}\tau_{lat}^3 \) with \( B = B(T) \)
- Lateral shear strain rate \( d\gamma_{lat}/dt \) increases as \( (\tau_{lat})^3 \) (Glen’s law); heating rate in the ice increases as \( (\tau_{lat})^4 \), \( \sim \) as \( x^4 \).

\[ \rightarrow \text{Heating source } \tau_{lat} (d\gamma_{lat}/dt) \text{ quickly becomes a significant heat source within the ice sheet and may induce internal melting at a large enough distance } x \text{ from the center} \]
Simple temperature profile at WAIS margins

• Steady state
• For simplicity we do not consider vertical advection of ice as:
  . ice accumulation at top and melting at base making the ice column colder
  . basal freeze-on making the ice column warmer

\[ K \frac{\partial^2 T}{\partial y^2} + \tau_{lat} \gamma = 0 \]

• \( \tau_{lat} \) and \( \gamma \) uniform in depth
• \( T(y=H) = T_{atm} \) and \( T(y = 0) = T_m \)

\[ T(y) = T_m + (T_{atm} - T_m) \frac{y}{H} + \frac{\tau_{lat} \gamma H^2}{K} \frac{y}{H} \left( 1 - \frac{y}{H} \right) \]

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Fig. 5: Location of margins temperature profile and force balance calculations.
Table: Ice streams characteristics from Joughin et al., J. Geophys. Res., 2002.

<table>
<thead>
<tr>
<th>Ice stream</th>
<th>$H$ (m)</th>
<th>$\dot{\gamma}_{lat} \ (10^{-2}\text{ yr}^{-1})$</th>
<th>$\tau_{lat} (\text{kPa})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1242</td>
<td>2.6</td>
<td>131.5</td>
</tr>
<tr>
<td>B</td>
<td>846</td>
<td>9.0</td>
<td>191.1</td>
</tr>
<tr>
<td>C</td>
<td>1805</td>
<td>0.7</td>
<td>90</td>
</tr>
<tr>
<td>D</td>
<td>888</td>
<td>3.2</td>
<td>134.2</td>
</tr>
<tr>
<td>E</td>
<td>916</td>
<td>6.3</td>
<td>171.9</td>
</tr>
</tbody>
</table>

$T(y) = T_m + (T_{atm} - T_m) \frac{y}{H} + \frac{\tau_{lat} \dot{\gamma}_{lat} H^2}{K} \frac{y}{H} \left(1 - \frac{y}{H}\right)$

Fig.: First approximation of ice stream A margins temperature. 
(assuming $T_{atm} = -20^\circ C$)
Simple temperature profile at WAIS margins

Predicted margin temperatures are in excess of melting over some depth range for all five profiles. This supports the possibility that internal melting within the ice sheet is related to why margins are where they are.
Approximate model of partially melted marginal zone: *Toward a more realistic temperature profile*

- $\& (\equiv \&_{lat})$ is assumed uniform for $0 < y < H$, 
  $\tau_{lat} = B(T)(\&/2)^{1/3}$.
- For $0 < y < H'$: $T = T_{melt} \approx 0^\circ C$
- For $H' < y < H$: $K \frac{d^2T}{dy^2} + \tau_{lat} \& \approx K \frac{d^2T}{dy^2} + 2B_{avg}(\&/2)^{4/3} = 0$, 
  where $B_{avg} \equiv B((T_{melt} + T_{atm})/2)$.

1/ Solution, $H' < y < H$: 
$$ K(T - T_{melt}) + B_{avg}(\&/2)^{4/3}(y - H')^2 = 0 $$

2/ $\Rightarrow K(T_{melt} - T_{atm}) = B_{avg}(\&/2)^{4/3}(H - H')^2$ (determines $H'$)
Fig.: Ice stream E margin temperature profile. Input parameters: $H$, $T_{melt}$, $T_{atm}$, $B_{avg}$ and shear strain rate.

For $H' < y < H$:
\[ K \frac{d^2T}{dy^2} + 2B_{avg}(\gamma/2)^{4/3} = 0, \]
where $B_{avg} \equiv B\left(\frac{T_{melt} + T_{atm}}{2}\right)$

For $0 < y < H'$: $T = T_{melt} \approx 0^\circ C$

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### Results of the approximate model of partially melted marginal zone

<table>
<thead>
<tr>
<th>Ice stream</th>
<th>$B_{avg}$ ($kPa \cdot yr^{1/3}$)</th>
<th>$\dot{\gamma}_{meas}$ ($10^{-2} \cdot yr^{-1}$)</th>
<th>$H'/H$</th>
<th>$\dot{\gamma}_{H'=0^+}$ ($10^{-2} \cdot yr^{-1}$)</th>
<th>$\dot{\gamma}<em>{meas} / \dot{\gamma}</em>{(H'=0^+)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>498</td>
<td>2.6</td>
<td>0.11</td>
<td>2.1</td>
<td>1.2</td>
</tr>
<tr>
<td>B</td>
<td>490</td>
<td>9.0</td>
<td>0.44</td>
<td>3.8</td>
<td>2.4</td>
</tr>
<tr>
<td>C</td>
<td>517</td>
<td>0.7</td>
<td>Not melted</td>
<td>1.2</td>
<td>0.6</td>
</tr>
<tr>
<td>D</td>
<td>498</td>
<td>3.2</td>
<td>Not melted but close</td>
<td>3.6</td>
<td>0.9</td>
</tr>
<tr>
<td>E</td>
<td>498</td>
<td>6.3</td>
<td>0.33</td>
<td>3.4</td>
<td>1.9</td>
</tr>
</tbody>
</table>

**Table**: Results of the approximate model of partially melted marginal zone.

Since lateral shear strain rate of ice stream D is really close to the level at which internal melting occurs, the results are compatible with the hypothesis that the margin of fast flowing ice is undergoing internal melting!
Force equilibrium gives us the average lateral shear stress \( \overline{\tau}_{lat} \):

\[
H \overline{\tau}_{lat} \approx H' B_{melt} (\text{gy} / 2)^{1/3} + (H - H') B_{avg} (\text{gy} / 2)^{1/3}, \quad \text{where } B_{melt} = B(T_{melt})
\]

\[
\overline{\tau}_{lat} = \left( \frac{K (T_{melt} - T_{atm}) B_{avg}^3}{H^2} \right)^{1/4} \frac{B_{melt}}{B_{avg}} \frac{H'}{H} \left( 1 - \frac{H'}{H} \right)^{1/2}
\]

For \( T = T_{avg} = -13^\circ C \) (i.e., \( T_{atm} = -26^\circ C, T_{melt} = 0^\circ C \)),

\( K = 2.32 \text{ W/m}^\circ C \) and \( B_{avg} = 498 \text{ kPa yr}^{1/3} \); also, \( B_{melt} = 247 \text{ kPa yr}^{1/3} \)

\( \overline{\tau}_{lat} \big|_{H' = 0^+} = 131.36 \text{ kPa} \)

\( \overline{\tau}_{lat} \big|_{H' = H/2} = 1 + \frac{B_{melt}}{B_{avg}} = 1.044 \quad \text{for ice stream D} \)

Recalling \( \overline{\tau}_{lat} \propto x \) (= distance from ice stream centerline):
3/ Massive drainage to the bed associated with melting causes channel development

*Melt generation and water permeation through the partially melted ice*


![Diagram of ice, water, and permeability](image1)

**Fig. 2.** Cross-section of a vein of liquid situated at a grain edge, where three grain boundaries meet. The figure is drawn for a dihedral angle $\phi$ equal to 20°, as measured for ice—water by Ketcham and Hobbs (1969).

![Diagram of water veins](image2)

**Fig. 3.** A junction between four water veins in polycrystal-line ice. The figure is a tetrahedron with non-spherical faces and with open corners.

Permeability $k = \alpha n^2 d_g^2$

$n = \text{porosity}, \ d_g = \text{grain size (1-10 mm in ice)}, \ \alpha \approx 1/2000 \ \text{to} \ 1/1500$

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**Darcy Law**: 
\[ \mathbf{r} q = - \frac{k}{\mu_w} (\nabla p - \rho_w \mathbf{r} g) , \quad k = \alpha n^2 d_g^2 \]

\[ \Rightarrow q_y = - \frac{\alpha n^2 d_g^2}{\mu_w} (\rho_w - \rho_i) g \quad \text{(assuming } \nabla p = \rho_i g) \]

**Melt production**: 
Melt rate per unit volume \( \mathbf{\dot{m}} = \tau_{\text{lat}} \& / L \), \( L = \text{latent heat} \), 
\[ \tau_{\text{lat}} = B \frac{2 B_{\text{melt}}}{L} \left( \frac{\&}{2} \right)^{1/3} F(n) \quad \text{with, e.g., } F(n) = 1 - n \]

**Mass conservation** \( \nabla (\rho_w \mathbf{r} q) = \mathbf{\dot{m}} \): 
\[ \rho_w \alpha (\rho_w - \rho_{\text{ice}}) g \frac{d}{dy} \left( n^2 d_g^2 \right) - \frac{2 B_{\text{melt}}}{L} \left( \frac{\&}{2} \right)^{4/3} F(n) = 0 \]

(we solve with \( d_g = \text{constant}, 2 \text{ or } 10 \text{ mm}, \) and \( \alpha = 1 / 1500 \); 
functions like \( F(n) = 1 - n^\lambda \), \( \lambda \geq 1 \), have negligible effect on results compared to \( F(n) = 1 \).)
3/ Massive drainage to the bed

Ice Stream B \((\bar{\tau}_{\text{lat}} = 135.13 \text{ kPa})\)

- Altitude (m): H = 850, H = 320
- Impermeable
- Permeable
- Bottom of ice
- \(T_{\text{atm}} = -20 \degree C\)
- \(T_{\text{melt}} = 0 \degree C\)
- \(q = 3.6 \text{ m.yr}^{-1}\)

Porosity

\(k = 2.3 \times 10^{-16} \text{ m}^2\)
3/ Massive drainage to the bed

Ice Stream B \((\bar{\tau}_{lat} = 145.95 \text{ kPa})\)

- **Altitude (m)**
  - \(H = 850\)
  - \(H' = 530\)

- **IMPERMEABLE**
- **PERMEABLE**

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- **Bottom of ice**
  - \(T_{atm} = -20 \degree C\)
  - \(T_{melt} = 0 \degree C\)

- **Porosity**
  - \(k = 6.2 \times 10^{-16} \text{ m}^2\)

- **Discharge**
  - \(q = 5.9 \text{ m yr}^{-1}\)

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8% more

1.6x more
Conclusions

• Our approximate model of margins predicts that margins are partially melted for 4 of the 5 ice streams in West Antarctica.

• Then, we argue that internal melting within the ice sheet is related to why margins are where they are.

• We predict meltwater production within the ice sheet at margins, and explore how it rains downward and creates a massive drainage to channelized drainage at margins bed.
$N_c = P_i - p_w = K_2 \frac{G^{11/24} Q_w^{1/12}}{n_m^{1/4}} B$

with

$K_2 = [(\rho_i L_f K_1)^{1/3} (\rho_w g)^{1/8}]^{-1}$

Using $Q_w = q_w \times 100 \ \text{km} \times 1 \ \text{km}$

we find $N_c = 0.4 \ \text{MPa}$

$\tau_{base} \approx 10 \ \text{kPa} = 0.01 \ \text{MPa}$

$\Rightarrow N \approx 0.02 \ \text{MPa}$ with $f=0.5$

• Effective stress $N_c$ on bed adjacent to the channels is much larger than $N$ in central region of the ice stream.

• $N_c$ locks ice sheet to the bed outboard of the R-channel and creates limit of the width of stream of fast-flowing ice.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_i$</td>
<td>Ice overburden pressure</td>
</tr>
<tr>
<td>$p_w$</td>
<td>Water pressure in channel</td>
</tr>
<tr>
<td>$G$</td>
<td>Force per unit volume driving water flow</td>
</tr>
<tr>
<td>$Q_w$</td>
<td>Water discharge in channel</td>
</tr>
<tr>
<td>$n_m$</td>
<td>Manning roughness coefficient</td>
</tr>
<tr>
<td>$B$</td>
<td>Creep parameter</td>
</tr>
<tr>
<td>$L_f$</td>
<td>Latent heat</td>
</tr>
<tr>
<td>$K_1$</td>
<td>Constant equal to 0.139</td>
</tr>
<tr>
<td>$q_w$</td>
<td>Darcy flux at bed margins because of ice internal melting</td>
</tr>
</tbody>
</table>