Verifying thermocoupled ice sheet models (and explaining the "warm spokes")

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Outline

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SIA flow versus "dragging ice shelves"

central fact for Shallow Ice Approximation = SIA:

Velocities are determined locally and depend on depth. Only horizontal plane shear stresses are included. h is surface elevation:

$$\langle \sigma_{xz}, \sigma_{yz} \rangle = \rho g (h - z) \nabla h$$

MacAyeal (1989) equations for ice streams:

Velocity is determined globally and is depth-independent. Stress balance includes only longitudinal and vertical plane shear stresses:

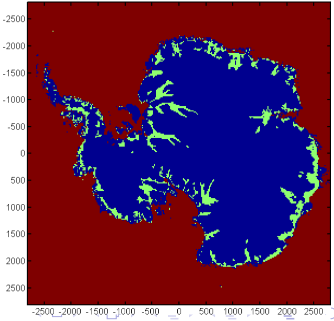
$$\left[2\nu H(2u_x + v_y)\right]_x + \left[\nu H(u_y + v_x)\right]_y - \rho g H h_x = \beta u$$
$$\left[2\nu H(2v_y + u_x)\right]_y + \left[\nu H(u_y + v_x)\right]_x - \rho g H h_y = \beta v$$

(effective viscosity ν depends on strain rates and temperature; H is thickness; β is basal drag)

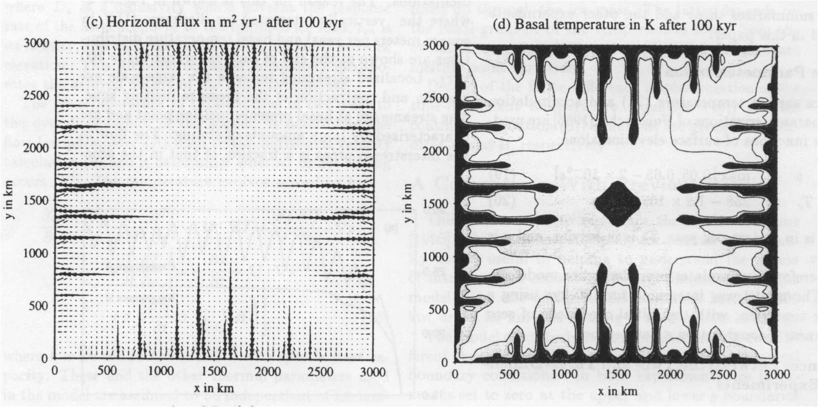
Antarctic sheet model with "mask" for flow type

- some equations apply everywhere (mass continuity and conservation of energy)
- but different areas get different computations of velocity; idea first successfully used in 3D Antarctic model by Ritz et al (2001)
- the whole model is still shallow; this is not the full Stokes' system

- **blue**: shallow ice approximation (SIA) computes velocity
- **green**: MacAyeal (1989) equations for dragging ice shelves compute velocity
- **red**: if there is ice, MacAyeal-Morland shelf equations compute velocity



But what is the status of these “ice streams”?



(Payne & Dongelmans 1997)

These “ice streams”, a.k.a. “spokes”, are fast flows of warm ice in the thermocoupled SIA itself on a non-sliding bed.

Outline

Everyone has the disease

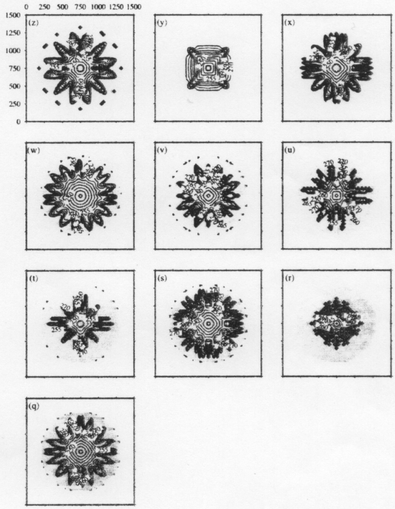


Fig. 3. Predicted steady-state basal temperatures in experiment F for each model in the intercomparison.

- these are basal temperature contour maps
- these spokes (in EISMINT II experiments) should not appear because they are numerical solutions to an angularly-symmetric continuum problem

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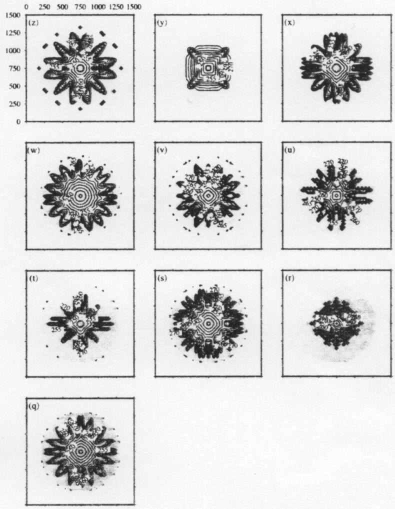

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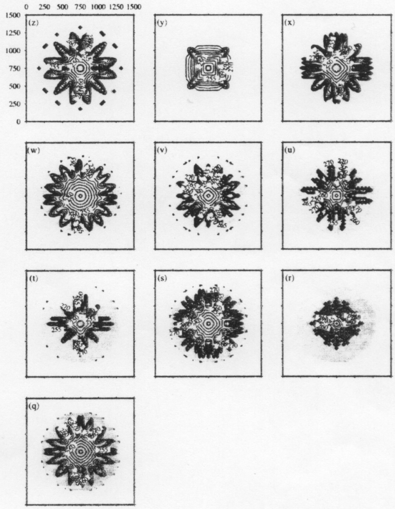

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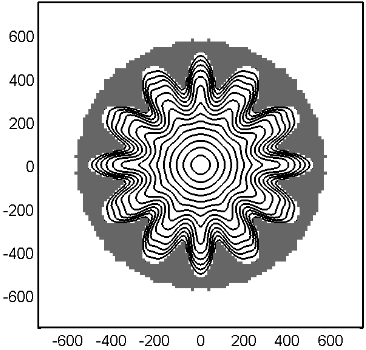
- Are they numerical errors? Yes!
- Are they just numerical errors? No. They are telling us something important about the continuum problem.

"Everyone" includes our group!

Yes, we get spokes when we run EISMINT II experiment F:



(grey=at pressure-melting temperature; $2\,K$ contour interval)

Moreover, who can trust ice sheet modelers?

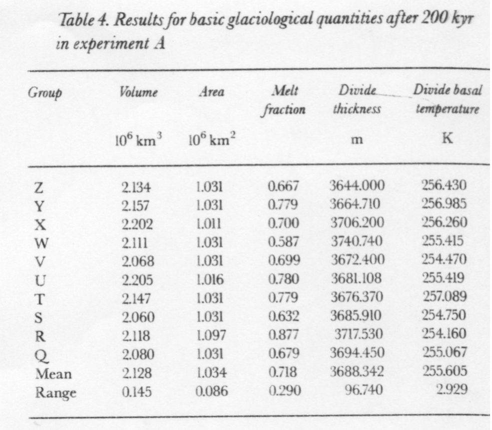


Table 4. Results for basic glaciological quantities after 200 kyr in experiment A

Group	Volume 10^6 km^3	Area 10^6 km^2	Melt fraction	Divide thickness m	Divide basal temperature K
Z	2.134	1.031	0.667	3644.000	256.430
Y	2.157	1.031	0.779	3664.710	256.985
X	2.202	1.011	0.700	3706.200	256.260
W	2.111	1.031	0.587	3740.740	255.415
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Mean	2.128	1.034	0.718	3688.342	255.605
Range	0.145	0.086	0.290	96.740	2.929

(table from EISMINT II = Payne et al 2000)

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- note wide range of results in "easy" experiment A
- what is a reliable way to estimate magnitude of numerical errors for a particular numerical model?
- it's not good enough to say "we matched EISMINT" ...

Outline

The context

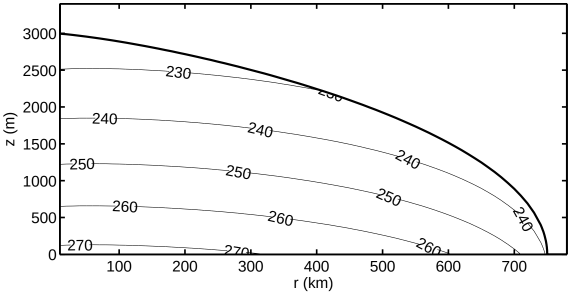
The problem

The new tools, and how to use them

The problem again, but in better focus

The future

Exact solutions to thermocoupled SIA



- new, simultaneous exact solutions to all equations in the thermocoupled SIA (i.e. mass continuity, flow law, incompressibility, and conservation of energy)

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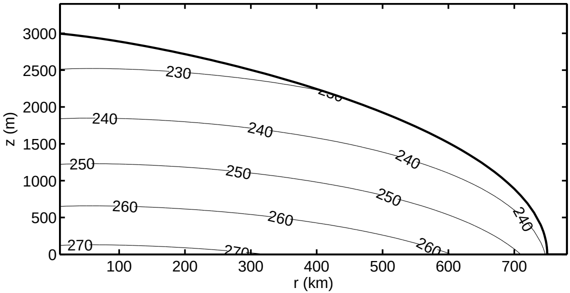

- new, simultaneous exact solutions to all equations in the thermocoupled SIA (i.e. mass continuity, flow law, incompressibility, and conservation of energy)
- no, I won't show you the formulas (codes are online, though)
- circular ice caps like EISMINT

Exact solutions: how to? (By analogy, anyway)

- completely made-up PDE:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u^2$$

is hard to find any exact solutions

- but one can find such for a slightly more general PDE:

$$\frac{\partial u}{\partial t} \overset{*}{=} \frac{\partial^2 u}{\partial x^2} + u^2 + f(x,t)$$

- for *example*, let $u(x,t) = x^3 + t$; compute

$$f = \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} - u^2 = 1 - 6x - \left(x^3 + t\right)^2$$

- with this f, equation $*$ has $u = x^3 + t$ as solution

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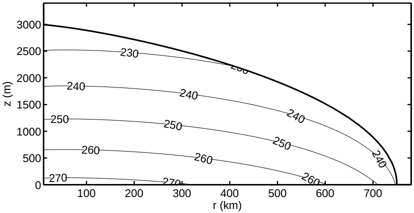
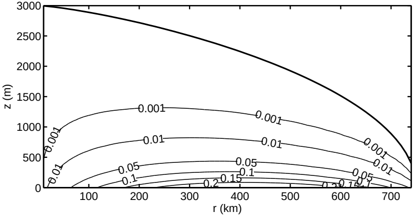
- So our exact solutions are for equations describing *radioactive* ice, more or less.
- But exact solutions found this way are really useful for:
    - checking correctness of numerical codes
    - getting some scale for achievable/reportable numerical error on a given grid

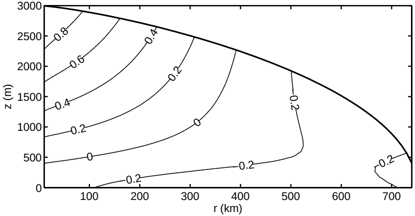
More views of exact solutions to thermocoupled SIA



left: exact profile and temperature

(*above*: contour labels in K; *below*: contour labels in $10^{-3}\,\mathrm{K\,a^{-1}}$)





left: exact strain-heating

right: exact added “radioactive” heating

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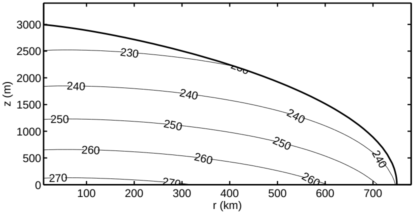

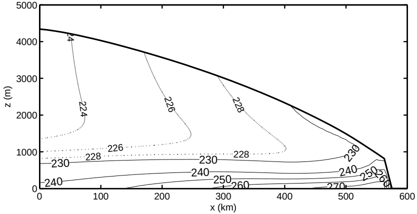

left: exact profile and temperature

right: computed EISMINT II experiment F

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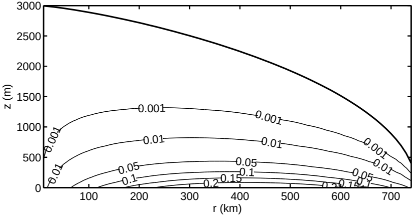

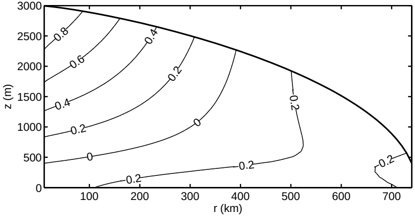

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- *verification* = solving the equations right
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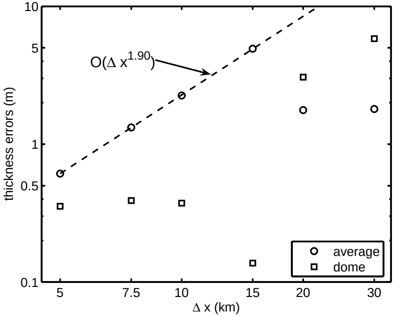
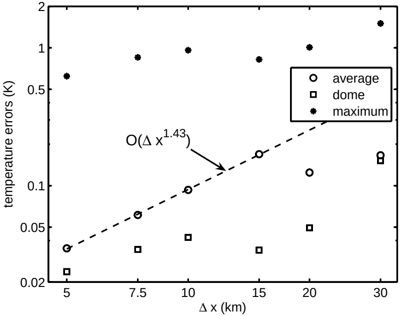
- *verification* = solving the equations right
- compare *validation* = solving the right equations (by comparison to real ice flow data!)
- "my numerical scheme has been verified" should mean:
  1. you know an exact (or very accurate) solution of the full mathematical model
  2. your numerical scheme approximates that mathematical model (and does not add additional physical guesses you made in each grid cell)
  3. you used the numerical scheme to approximate the exact solution
  4. you know how big the error is in that computation
  5. you show that the error decreases as the grid is refined

Convergence under grid refinement

Because we know exact solutions to thermocoupled SIA,

- we know size of actual numerical errors and
- convergence rate under grid refinement can be measured





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- the verification process can clarify some mysteries . . .

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An update on the spokes

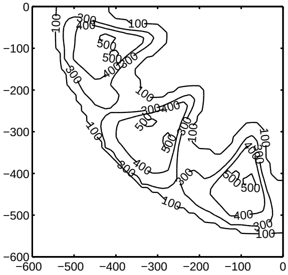
- we still get spokes after we verify the scheme (for EISMINT II experiment F)
- spokes reflect a sensitivity of the continuum equations to perturbation in some geometry/temperature regimes; compare (Hindmarsh 2004)

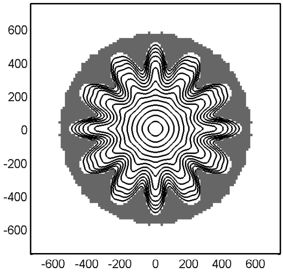
What causes the spokes

careful error analysis of our finite difference scheme for the temperature equation clearly identifies

the derivative with respect to temperature of the strain-heating term

as the controlling quantity in the spokes





left: $\partial(\text{strain-heat})/\partial T$ in $10^{-12}\,\text{s}^{-1}$; *right*: basal temperature

The bad news

- the thermocoupled SIA really is subject to spokes: the coupled continuum equations are very sensitive to perturbation
- perturbations cannot be avoided when doing numerics!
- the continuum system might even be ill-posed in some geometry/temperature regimes (Hindmarsh 2006)
- *prediction*: the full thermocoupled Stokes' equations also have this kind of sensitivity to perturbation

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- codes will be massive and parallel and coupled to other climate systems and really confusing: include verification in ice sheet codes when possible!
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- QUESTIONS?