Better physics using full momentum solver in 2D vertical slice domain, where does longitudinal stress really matter? Application to the Thwaites Glacier flowline

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Introduction:

Shallow-Ice Approximation:

The commonly used shallow-ice approximation neglects all stresses except the basal drag, an assumption that is very good for inland ice but may be very poor for fast-flowing, low-surface slope ice streams, where longitudinal stresses may not only be important, but may in fact be the dominant stress [2].

Higher-Order Approach:

A higher-order approach is to couple the mass- and momentum-conservation equations (the prognostic and diagnostic equations [6]) and solve with no neglected stresses. In the process of developing such a full-momentum solver in 3D, for embedded application within the map-plane University of Maine Ice Sheet Model (UMISM) [1], we are testing a 2D simplification which models a vertical slice through the ice sheet along a flowline.

Allows Us To:

This allows us to do two things: 1) implement and test the complex boundary conditions that must be specified for a full-momentum solver, and 2) evaluate when and where longitudinal stresses are important or even dominant.

The Differential Equation:

Conservation of Momentum:

Conservation of momentum (ie. the balance of forces) leads to the following equation in terms of the stress tensor σ and the various components of the body acceleration a:

$$\sigma_{ij,j} + \rho a_i = 0 \tag{1}$$

Stress components (σ) are related to strain rates ($\dot{\epsilon}$) through the usual flow law, where P is the pressure, μ is an effective viscosity, and δ_{ij} is the Kronecker-delta (1 when $i = j$, 0 otherwise).

$$\sigma_{ij} = \delta_{ij} P + 2\mu\dot{\epsilon}_{ij} \tag{2}$$

The non-linearity of the flow law requires that the effective viscosity μ depend on the strain rate invariant $\dot{\epsilon}$ in the following form.

$$2\mu = B\dot{\epsilon}^{\frac{1-n}{n}} \tag{3}$$

The strain rate invariant is given by the following.

$$\dot{\epsilon}^2 = \frac{1}{2}\dot{\epsilon}_{ij}\dot{\epsilon}_{ij} \tag{4}$$

Expressing the strain rates in terms of the velocities

$$\dot{\epsilon}_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{5}$$

The differential equation is then

$$\left(\delta_{ij} P + 2\mu\frac{1}{2}(u_{i,j} + u_{j,i})\right)_{,j} + \rho a_i = 0 \tag{6}$$

The usual FEM Galerkin approximation and variational leads to the following matrix equation

$$(K_{mn} + \Delta K_{mn})U_n = F_m \tag{7}$$

The Continuity Equation

Conservation of mass leads to a continuity equation,

$$\frac{\partial h}{\partial t} + (\bar{u}_i h)_{,i} = \dot{a} \tag{8}$$

where $\bar{u}$ is the column-average of the x and y components of the velocity field, h is the ice thickness, and $\dot{a}$ is the net mass balance at a point. This too can be solved with the FEM, again as a matrix difference equation

$$\left(\frac{C_{mn}}{\Delta t} + K_{mn}\right) h_n^{i+1} = F_m + \frac{C_{mn}}{\Delta t} h_n^i \tag{9}$$

Boundary Conditions:

Specification of the differential equation describing conservation of momentum (also referred to as the "balance of forces") allows for two types of boundary conditions [4]: 1) Dirichlet, where the state variable, in this case the velocity, is specified, and 2) Neumann, where the conserved flux, in this case the force applied on the boundary, is specified. Where the bed is frozen, Dirichlet boundary conditions are the obvious choice, as the velocity is zero and can be specified as such. Where the bed is not frozen, where sliding is occurring (for example, in ice streams, where our shallow-ice approximation breaks down), we cannot specify the velocity, but instead must specify the force exerted on the ice by the bed in resisting its forward motion.

Problems Specifying:

We know that this resistive force cannot exceed the driving stress (if it equals the driving stress, we have the shallow-ice solution). A temptation is to use some fraction of the driving stress, and indeed, this approach does produce the concave profile characteristic of an ice stream, but the fraction is hard to define (a model parameter).

A Boundary-Layer Approach:

A better approach, and the one we have taken, is to use a boundary-layer. We can preserve our Dirichlet-type specification of zero velocity on the boundary, but allow greater deformation within the boundary layer to simulate sliding at the bed. This soft layer can be interpreted either as a deformable till or as a layer of water-saturated ice at the melting point (slush). In either case its thickness will be negligible compared with the ice thickness, and while the geometry and mechanical properties (how thick and how soft) are still difficult to define, at least they have a physical meaning, which is a good thing for a model parameter to have.

Application to Thwaites Glacier:

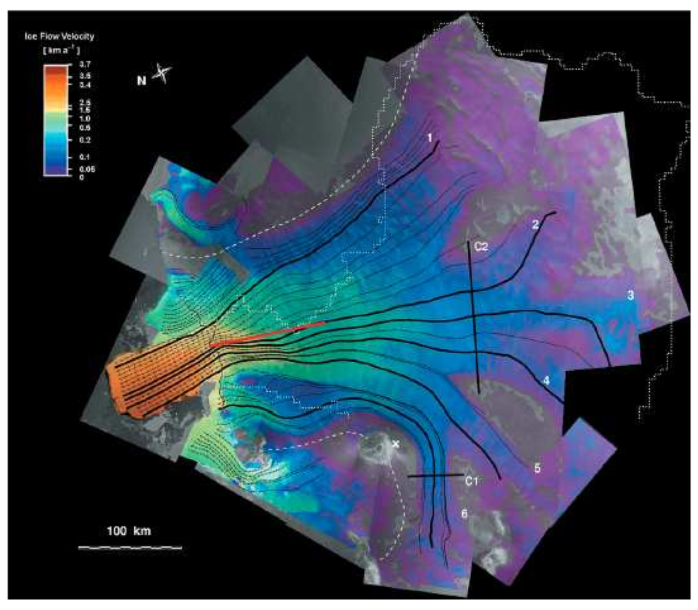
We apply this to a flowline along the Thwaites Glacier in the Amundsen Sea sector using excellent new data from the Airborne Geophysical survey of the Amundsen Sea Embayment (AGASEA), by University of Texas [3] and British Antarctic Survey [7] teams.

Description of Experiment

The first experiment shows the magnitude of the velocity and the ratio of longitudinal to vertical shear stress for a sequence of increasing area of decoupling of the ice to the bed. The sliding layer is progressively softened starting at a point a specified percentage of the distance up the flowline, with maximum softening attained at the grounding line. Percentages greater than 100% imply that the softening started outside the modeled area.

In the second experiment we show the stress ratio, the velocity magnitude, the velocity in the X direction, the vertical shear stress, and the longitudinal stress for cases both with and without Pulling Power. Here Pulling Power represents the longitudinal stress due to the imbalance of pressures within the ice and water columns at the calving ice front, assumed to be transmitted through the ice shelf to the grounding line.

Thwaites Velocity Data [5]:



References

[1] J.L. Fastook. The finite-element method for solving conservation equations in glaciology. *Computational Science and Engineering*, 1(1):55–67, 1993.

[2] J.L. Fastook and A. Sargent. Better physics in embedded models. In *Eleventh Annual West Antarctic Ice Sheet Initiative Workshop*, Sterling, Virginia, 2004.

[3] J.W. Holt, D.D. Blankenship, D.L. Morse, D.A. Young, M.E. Peters, S.D. Kempf, T.G. Richter, D.G. Vaughan, and H.F.J. Corr. New boundary conditions for the West Antarctic Ice Sheet: Subglacial topography of the Thwaites and Smith Glacier catchments. *Geophys. Res. Lett.*, L09502(doi:10.1029/2005GL025561), 2006.

[4] Thomas J.R. Hughes. *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*. Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1987.

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[6] D.R. MacAyeal. *EISMINT: Lessons in Ice-Sheet Modeling*. University of Chicago, Chicago, Illinois, 1997.

[7] D.G. Vaughan, H.F.J. Corr, F. Ferraccioli, N. Frearson, A. O'Hare, D. Mach, J.W. Holt, D.D. Blankenship, D.L. Morse, and D.A. Young. New boundary conditions for the West Antarctic Ice Sheet: Subglacial topography beneath Pine Island Glacier. *Geophys. Res. Lett.*, doi:10.1029/2005GL025588, 2006.

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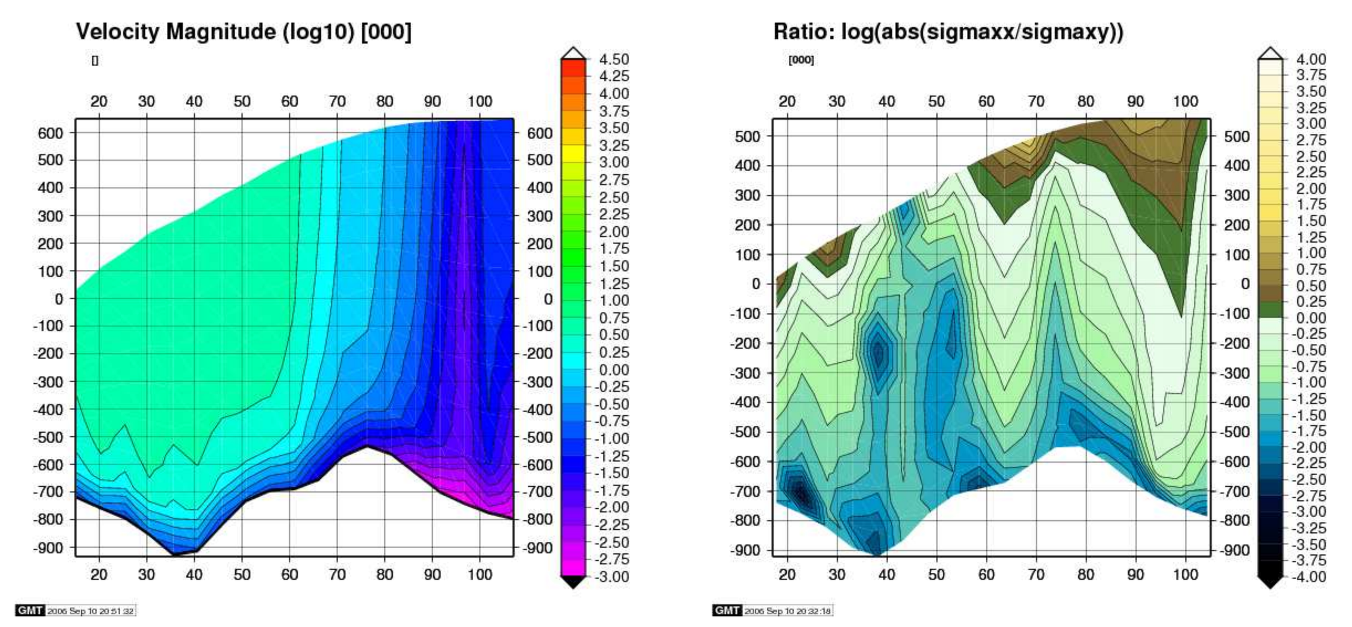
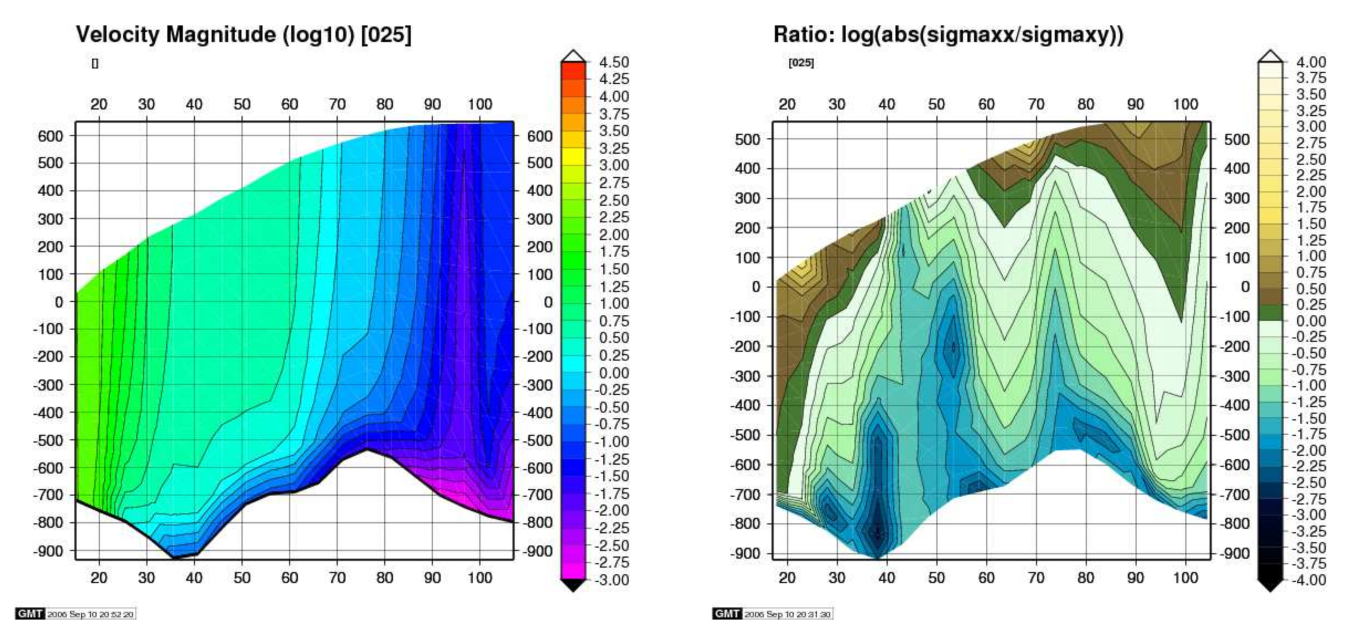
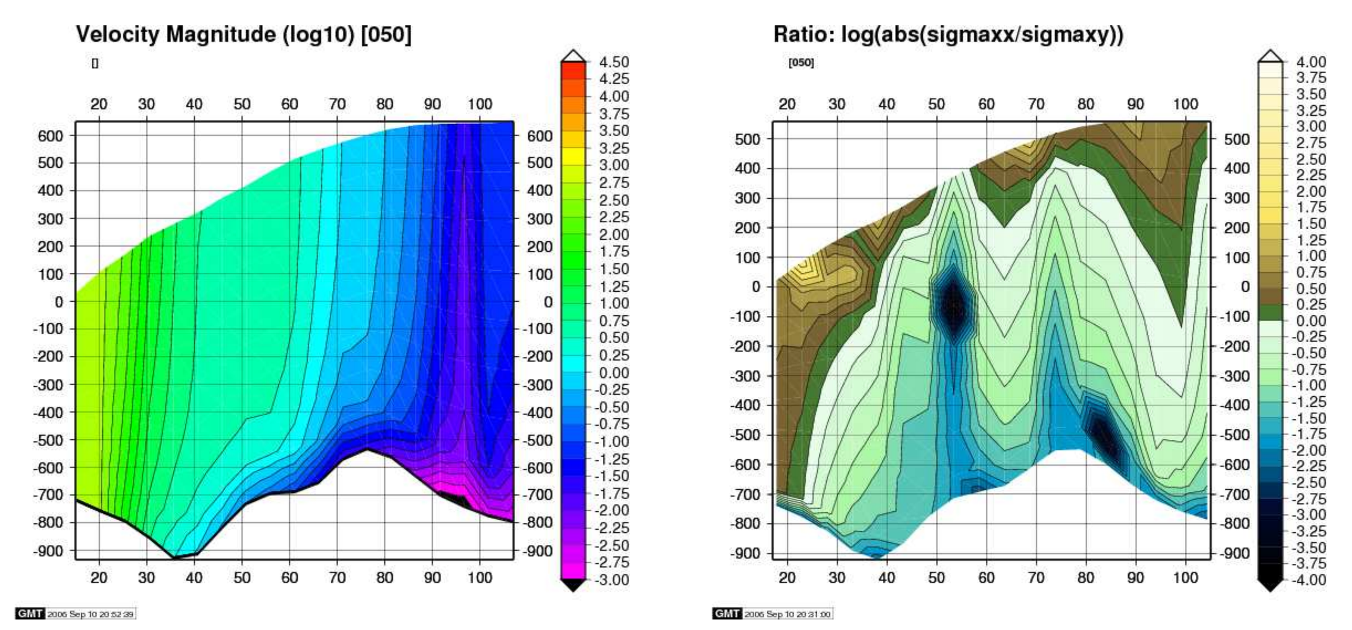
No Sliding, velocity magnitudes and stress ratios



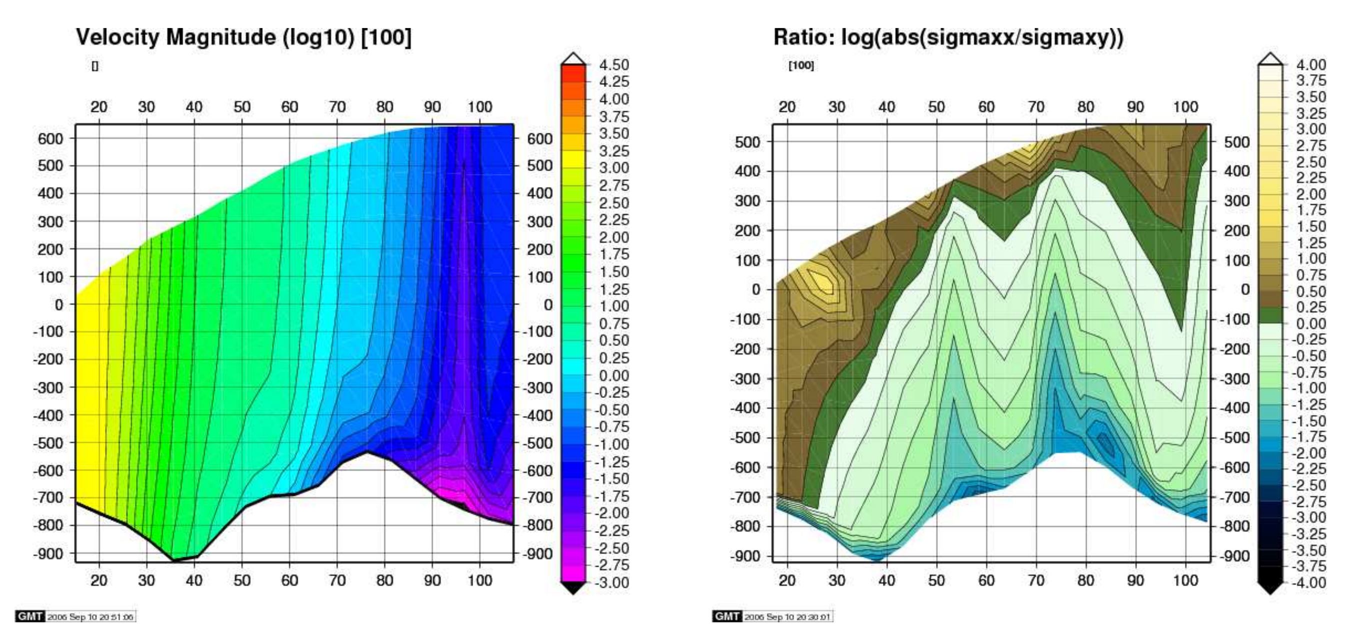
Decoupling begins at 25% of flowline.



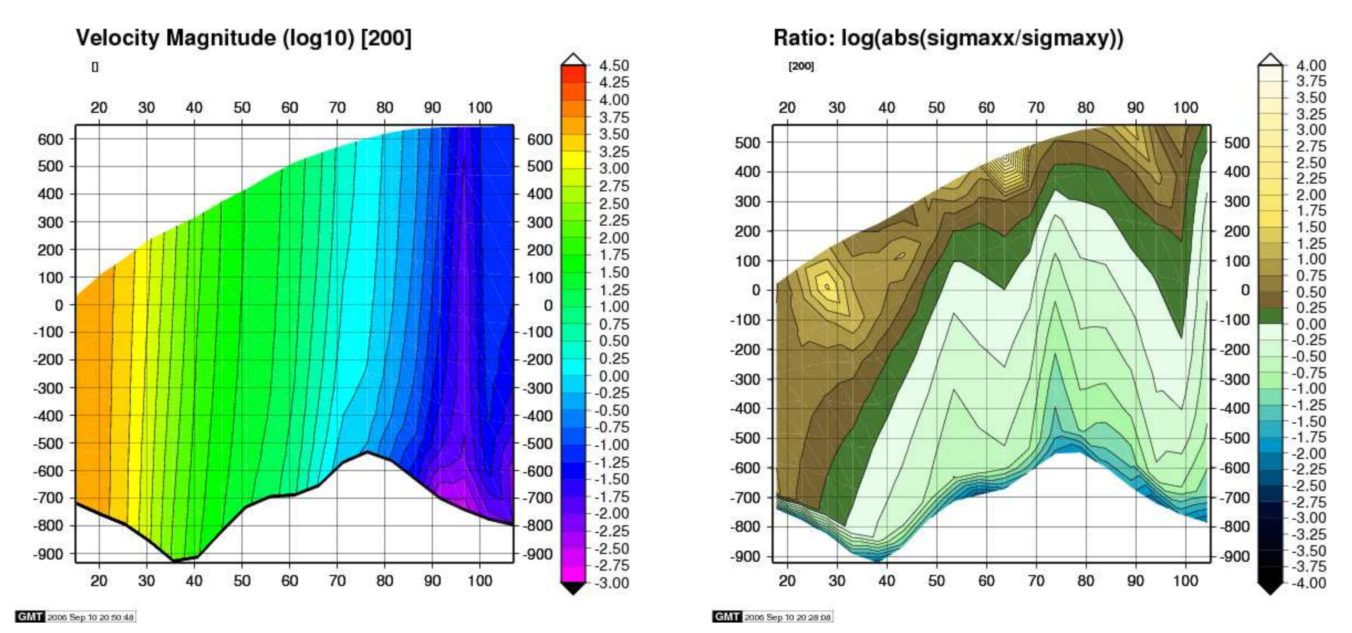
Decoupling begins at 50% of flowline.



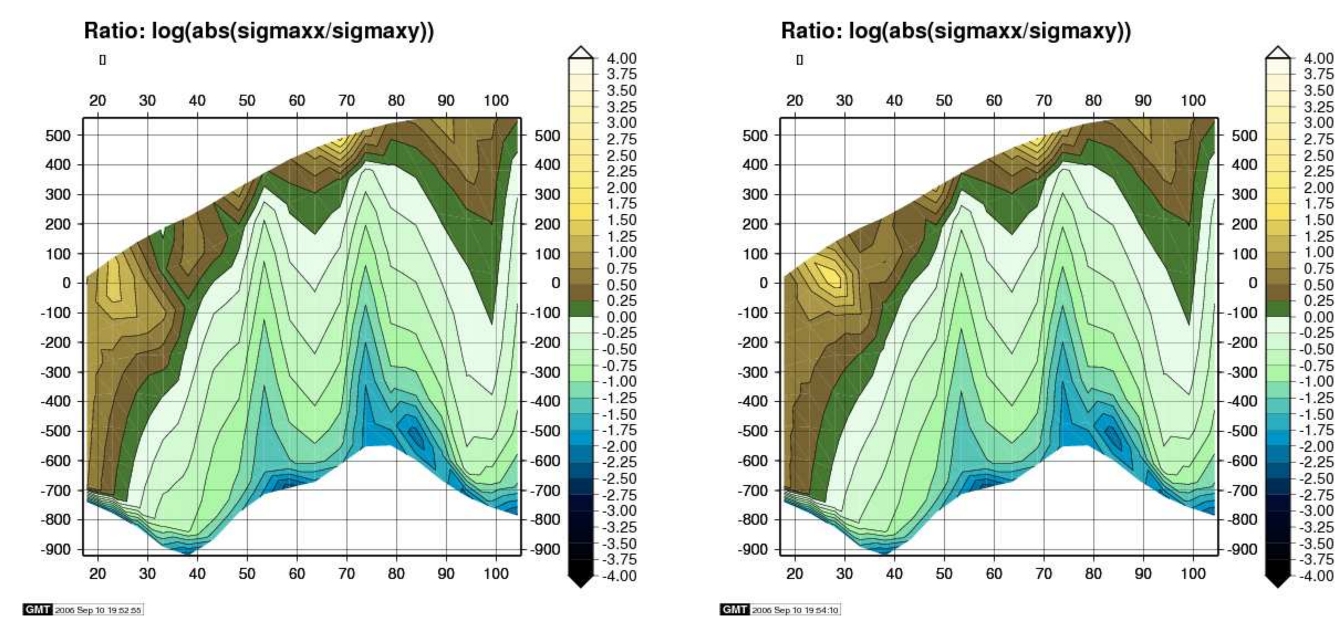
Decoupling begins at 100% of flowline.



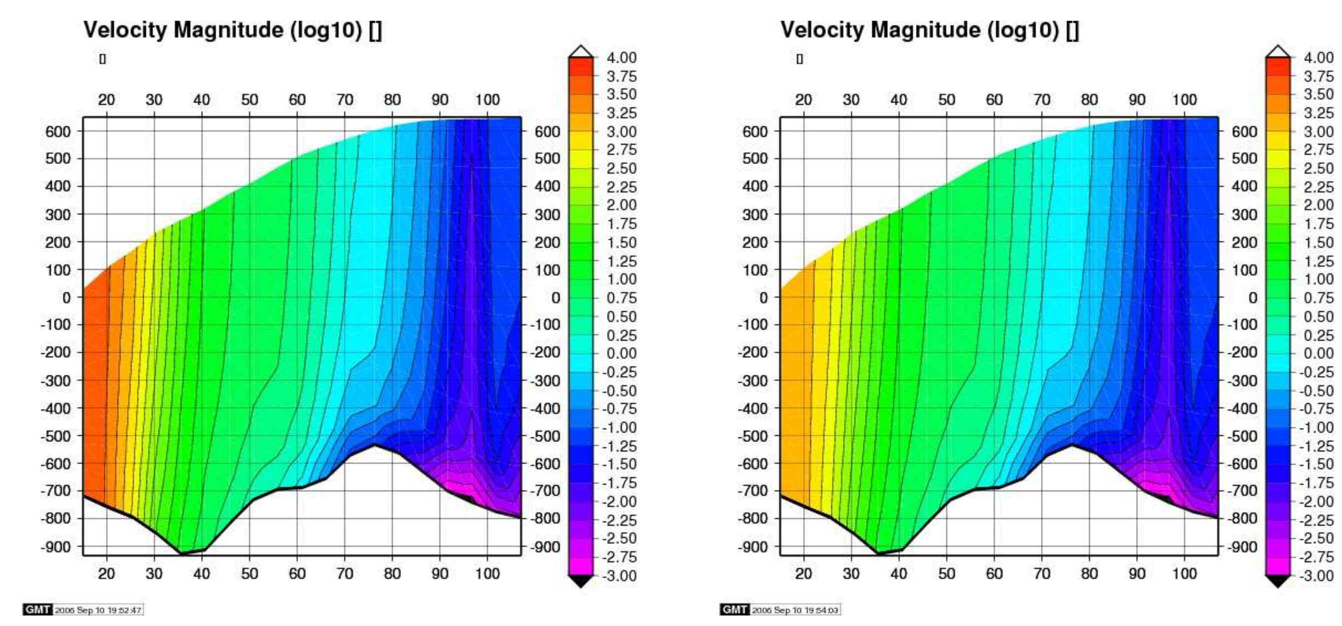
Decoupling begins at 200% of flowline.



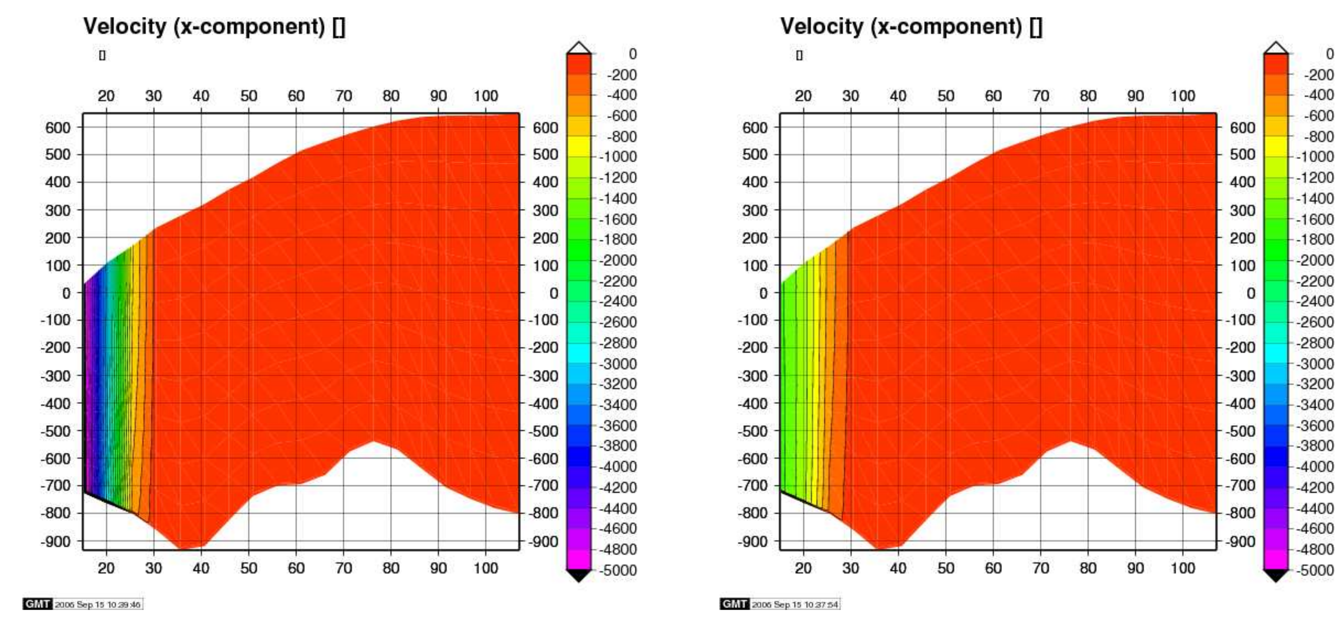
Stress ratios with and without Pulling Power



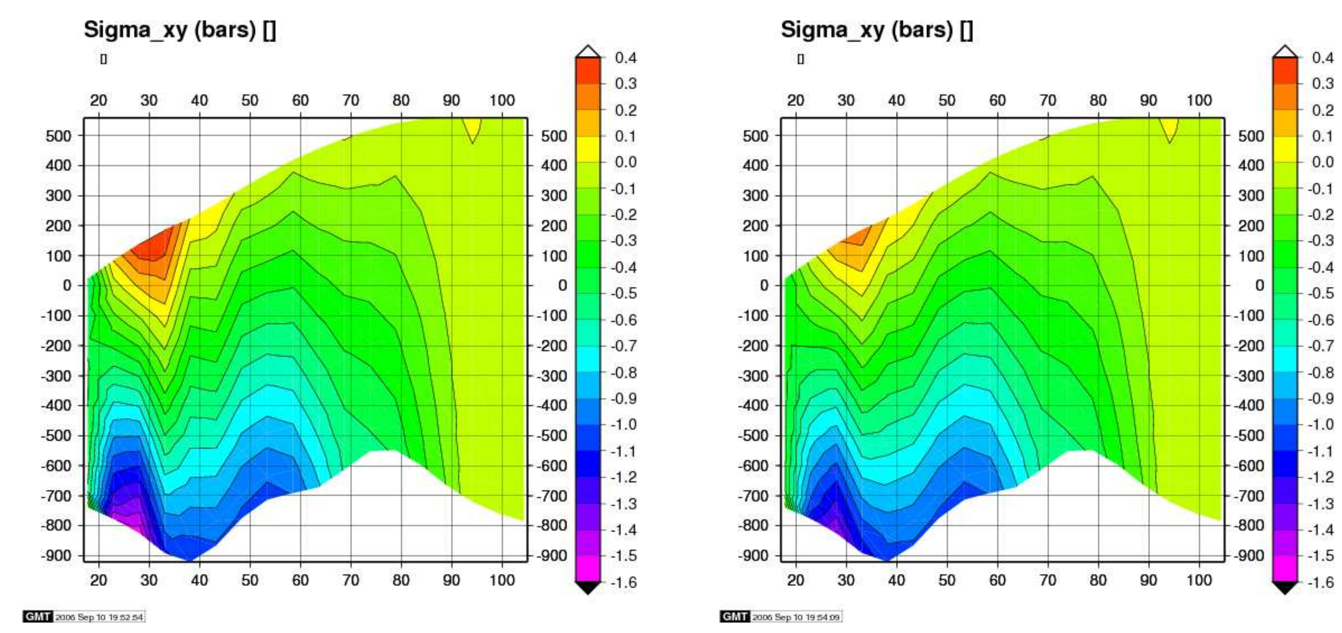
Log-10 of velocity magnitudes (m/yr)



X-Velocity (m/yr)



Vertical shear stress (bars), σ_{xy}



Longitudinal stress (bars), σ_{xx}